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FAILURE PREDICTION FOR AN ON-LINE MAINTENANCE SYSTEM IN A POISSON SHOCK ENVIRONMENT

by

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Failure Prediction for an On-Line Maintenance Systems in a Poisson Shock Environment

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TABLE OF CONTENTS

ABSTRACT

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CHAPTER 1

INTRODUCTION

Fault analysis processes have been and will continue to be very significant factors in the safety and reliability of electrical systems. This is especially true due to the following facts: a rapid advancement in the complexity and size of modern systems; increased availability and capabilities of computers; and rapidly changing technologies in integrated circuit fabrication. Due to this, fault analysis has become much more than an academic research topic. Fault analysis is applicable in an industrial environment to minimize cost, extend the lifetime of the overall system, control maintenance schedules, and effectively plan manpower needs.

The techniques of fault analysis (detection, location and prediction) have been developed independently in two different areas, digital and analog systems. In the last decade, considerable fault location research has been performed in the digital system area. Preparata, Metze and Chien (1) introduced an elegant graphical model and defined the properties of t-diagnosable systems. This model proved to be very powerful with the development of characterization theorems for the system by Hakimi and Amin (2, 3). In 1973, Russell and Kime (4) extended the Preparata-Metze-Chien

model into a more general case. However, such a generalization increases the complexity of the system.

Fault detection research for digital systems was introducted by Patterson and Metze (6) in 1972 and by Pradhan and Reddy (7) in 1973. Several different methods were developed for solving the problem of real-time fault detection in an asynchronous sequential circuit. After 1974, Maki and Sawin III (8, 9, 10) published a series of papers discussing the fault detection and fault-tolerant design for both synchronous and asynchronous sequential circuits.

The use of graphical models for dealing with analog circuits was first suggested by Kirchhoff in 1847 (11). Using Kirchhoff's model, Berkowitz (12) developed his concept of solvability for fault diagnosis in 1960. From the necessary conditions for solvability, a new concept, Key Subgraph (13), was introduced. Gayer (14) applied the Key Subgraph concept to fault isolation in simple linear solid state amplifiers.

Another aspect in the fault analysis process for an analog device is fault prediction. To accurately predict a fault, a device must be tested at periodic maintenance intervals. If the device fails or does not operate correctly, it is replaced immediately. The device may be assumed good if its characteristics are normal. However, if the characteristics are slightly out of tolerance, but

the device still operates correctly, one can attempt to predict if the device will fail before the next scheduled maintenance interval. If device failure is predicted, it can be replaced before failure occurs as part of planned preventive maintenance.

With the advent of the low-cost microprocessor, on-line fault prediction is possible and practical. A curve fitting algorithm for on-line fault prediction was first introduced by Saeks, Liberty and Tung (15, 16, 17) in 1975. It was assumed that prior life-time statistics for the system under test were known. Also, performance data of the system at each maintenance interval were collected. The application of these data to a second order polynomial equation resulted in an estimation of the time at which the component under test would exceed tolerance limits. Based on a criterion of simultaneously minimizing on-line failures and maximizing component lifetime, a decision as to whether or not the component should be replaced is made at each maintenance interval.

The disadvantages of this curve fitting algorithm are: the application is limited to failures due to permanent overstress, the second order polynomial is too simple to describe the performance of the component, and the prior lifetime statistics for the component are often not available.

Another area where an extensive research effort is being made is shock models and wear processes. Esary, Mar-

shall and Proschan (21, 22, 23) introduced a shock model for the life distribution of a component subjected to a sequence of shocks randomly occurring in the time according to a homogeneous Poisson process. They also considered the related shock models in which each shock caused a random amount of damage and failure occurred when the accumulated damage exceeded a specified threshold. This failure model is well known in modern reliability theory.

Employing the Poisson-Shock model, another curve fitting fault prediction algorithm which will overcome the disadvantages of the Saeks-Liberty-Tung algorithm will be discussed in the succeding chapters. In Chapter 2, an analog system subjected to the Poisson Shock is modeled using past performance data of the analog system. Algorithms for fault prediction in an on-line maintenance process are described. Several sequential refinement schemes have been developed to improve fault prediction. In Chapter 3, a decision rule is developed which is based on the criterion of simultaneously maximizing lifetime and minimizing the cost of on-line failures. Simulations of the proposed on-line maintenance process are presented in Chapter 4.

CHAPTER 2

SYSTEM MODELING

2.1 Failure Dynamics

The performance of an analog device subject to a series of discrete shocks (switching process, improper operation, etc...) may drift due to the shock damage. Let C (N) represent values of a particular component parameter, where the component time, N, denotes the number of shocks the component has received. In order to normalize the value of parameter C, a 1 or 0 is assigned to a "brand new" or "fail" stage respectively.

The normalized value of the parameter should have the following properties:

$$
\begin{array}{ll}
\text{(I)} & C(0) = 1 \\
\text{(II)} & C(\infty) = 0 \\
\text{(III)} & \text{If } L \text{ is the smallest integer that } C(L) = 0\text{, then} \\
& C(N) = 0 \ \forall\, N \geq L .
\end{array}
\qquad (2.1\text{-}1)
$$

It is assumed that drifting parameters can be well described by a difference equation of the following form:

$$C(N + 1) = C(N) - f(N) \qquad N < L \quad (2.1\text{-}2)$$

where f(N) is the particular failure dynamics.

Due to the simplicity of the calculation, it is practical to assume that f(N) has a polynomial form with order h. That is, eq. (2.1-2) can be rewritten as

$$C(N) - C(N+1) = a_0 + a_1 N + \ldots\ldots + a_h N^h = \sum_{i=0}^{h} a_i N^i \qquad (2.1\text{-}3)$$

where N < L and the $\{a_i\}$ are constants and L is the smallest integer satisfying the condition

$$\sum_{j=0}^{L-1} \sum_{i=0}^{h} a_i j^i \geq 1 \qquad (2.1\text{-}4)$$

Solving the difference eq. (2.1-3) with the boundary conditions (2.1-1), one obtains

$$\begin{aligned} C(N) &= 1 - \sum_{j=0}^{N-1} \sum_{i=0}^{h} a_i j^i \qquad \text{if } N < L \\ C(N) &= 0 \qquad \text{if } N \geq L \end{aligned} \qquad (2.1\text{-}5)$$

L is termed the "life-time" of the component, that is, the number of shocks necessary to cause the component to fail.

Consider a simple example where f(N) is taken to be a first order polynomial; that is,

$$f(N) = C(N) - C(N+1) = a_0 + a_1 N \qquad (2.1\text{-}6)$$

From equation (2.1-6) and boundary conditions (2.1-1), C(N) can be expressed as

$$C(N) = 1 - a_0 N - \frac{N(N-1)}{2} a_1 \quad \text{if } N < L$$
$$C(N) = 0 \quad \text{if } N \geq L \tag{2.1-7}$$

Then the life-time of this component is the smallest integer satisfying the equation

$$1 - a_0 L - \frac{L(L-1)}{2} a_1 \geq 0 \tag{2.1-8}$$

That is,

$$L \geq \frac{\sqrt{(2a_0 - a_1)^2 + 8a_1} - (2a_0 - a_1)}{2a_1} \tag{2.1-9}$$

Table (2.1-1) gives the life-time of the component, *satisfying eq.* (2.1-8), with different values of a_0 and a_1. Fig. (2-1-1) shows the life-time curve as a function of a_0 with different a_1's.

Table (2.1-1)

Component's Lifetime with different values of a_0 and a_1

L (a_0 \ a_1)	0.001	0.002	0.003	0.005	0.007	0.01	0.02	0.03	0.05	0.07	0.1
0.001	>30	>30	26	21	18	15	11	9	7	6	5
0.002	>30	>30	26	21	18	15	11	9	7	6	5
0.003	>30	>30	26	20	17	15	11	9	7	6	5
0.005	>30	30	25	20	17	15	11	9	7	6	5
0.007	>30	29	25	20	17	14	11	9	7	6	5
0.01	>30	28	24	19	17	14	11	9	7	6	5
0.02	30	24	21	17	15	13	10	8	7	6	5
0.03	25	21	19	16	14	12	10	8	7	6	5
0.05	18	16	15	13	12	11	9	8	6	6	5
0.07	14	13	12	11	10	10	8	7	6	5	5
0.1	10	10	9	9	9	8	7	6	6	5	5

$$L \geq \frac{\sqrt{(2a_0 - a_1)^2 + 8a_1} - (2a_0 - a_1)}{2a_1}$$

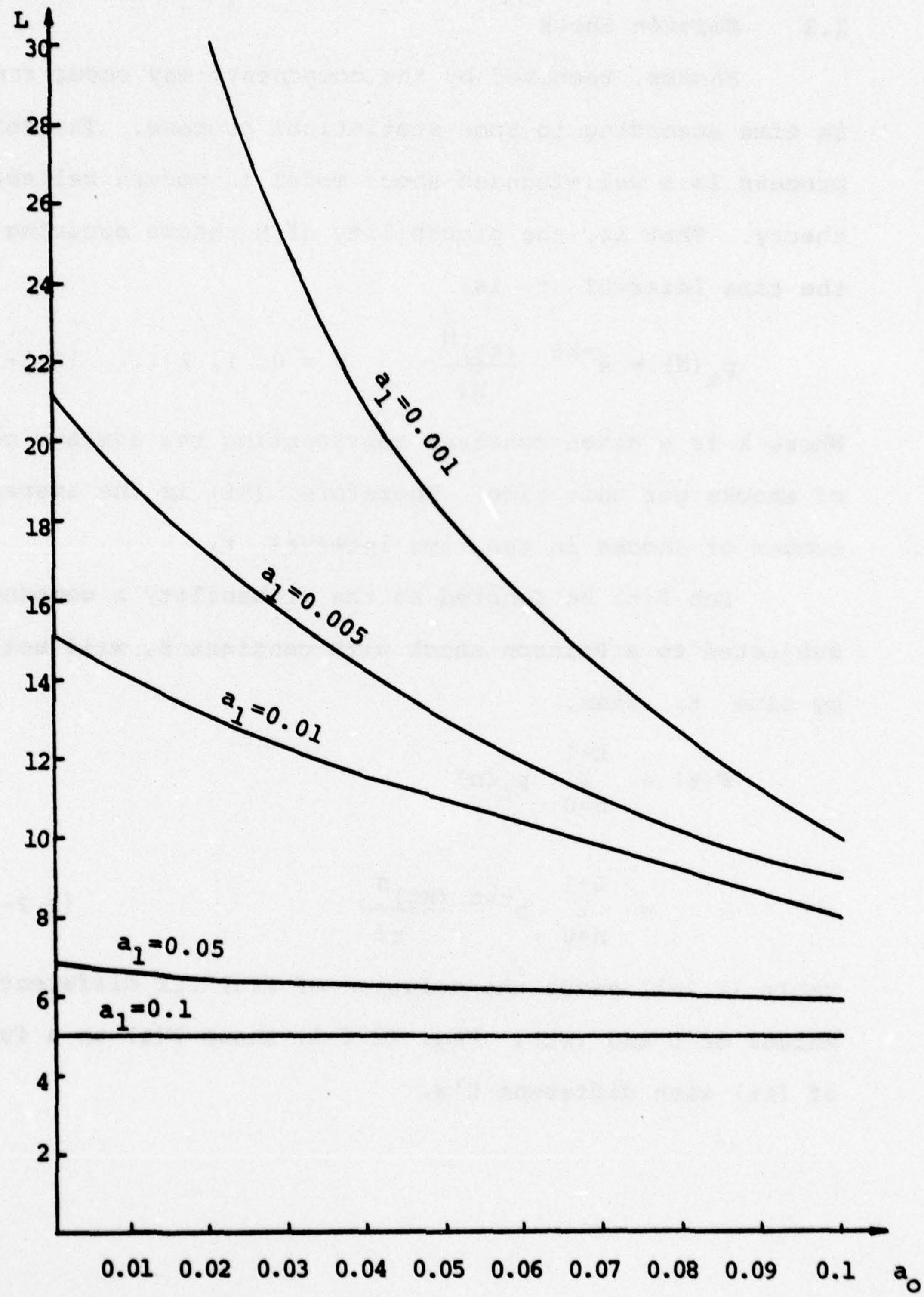

Fig. (2-1-1) Lifetime L v.s. a_0 with different values of a_1

2.2 Poisson Shock

Shocks, received by the component, may occur randomly in time according to some statistical process. The Poisson process is a well-founded shock model in modern reliability theory. That is, the probability of N shocks occuring in the time interval t is:

$$p_t(N) = e^{-kt} \frac{(kt)^N}{N!} \qquad N = 0, 1, 2 \ldots \qquad (2.2\text{-}1)$$

Where k is a given constant representing the average number of shocks per unit time. Therefore, (kt) is the average number of shocks in the time interval t.

Let F(t) be denoted as the probability a component subjected to a Poisson shock with constant k, will not fail by time t. Then,

$$F(t) = \sum_{n=0}^{L-1} p_t(n)$$

$$= \sum_{n=0}^{L-1} e^{-kt} \frac{(kt)^n}{n!} \qquad (2.2\text{-}2)$$

Table (2.2-1) gives the solution of F(t) for different values of L and (kt). Fig. (2-2-1) shows F(t) as a function of (kt) with different L's.

TABLE (2.2-1)

Solutions of F(t) for different values of L and (kt)

F(t)% / kt / L	1	2	3	4	5	6	7	8	9	10	11	12	15	20	30
2	74	41	20	9	4	2	0.7	0.3	0.1	0	0	0	0	0	0
3	92	68	42	24	12	6	3	1	0.6	0.3	0.1	0	0	0	0
4	98	86	65	43	27	15	8	4	2	1	0.5	0.2	0	0	0
6	100	98	92	79	62	45	30	19	12	7	4	2	0.3	0	0
8	100	100	99	95	87	74	60	45	32	22	14	9	2	0.1	0
10	100	100	100	99	97	92	83	72	59	46	34	24	7	0.5	0
12	100	100	100	100	99	98	95	89	80	70	58	46	18	2	0
15	100	100	100	100	100	100	99	98	96	92	85	77	47	10	0.1
20	100	100	100	100	100	100	100	100	100	100	99	98	88	47	2
25	100	100	100	100	100	100	100	100	100	100	100	100	99	84	16
30	100	100	100	100	100	100	100	100	100	100	100	100	100	98	48

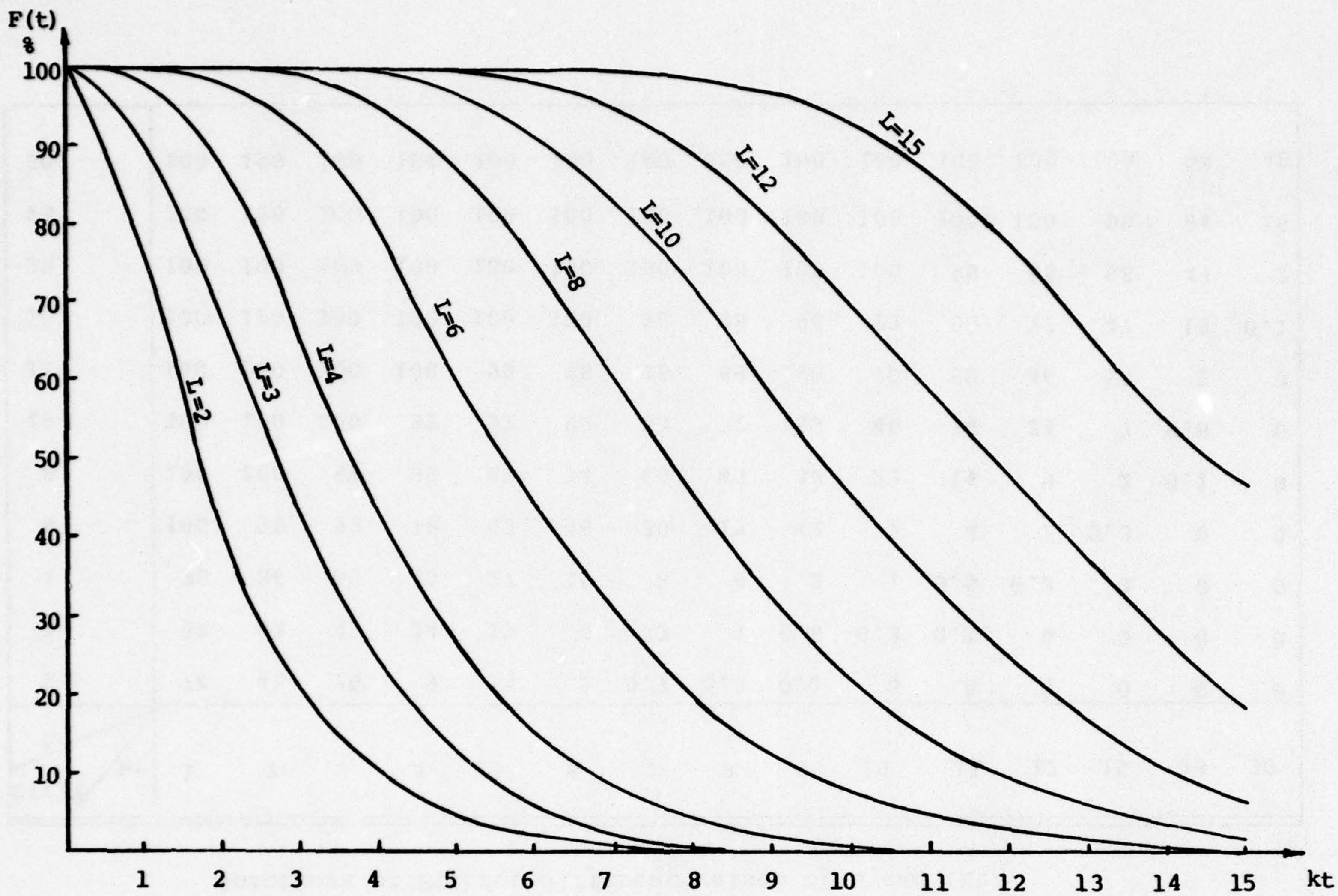

Fig. (2-2-1) F(t) v.s. (kt) with different lifetime

2.3 Estimation of the Failure Dynamics

In a periodic maintenance system, the performance data of a component is taken at each maintenance interval nT. Thia is, (C_1, C_2 ...., C_g) is the performance data taken at maintenance times (T, 2T ...., gT). The problem can be states as:

"Given performance data (C_1, C_2 ...., C_g), solve for the constants (a_0, a_1, ..., a_h) of the failure dynamics in eq. (2.1-3)."

Since it is assumed that the system is subjected to Poisson Shock with constant k, that is the average number of shocks in each time interval T is kT. Therefore, eq. (2.1-5) can be rewritten as:

$$\sum_{j=0}^{mkT-1} a_0 j^0 + \sum_{j=0}^{mkT-1} a_1 j^1 + \ldots. + \sum_{j=0}^{mkT-1} a_h j^h = 1 - C_m \tag{2.3-1}$$

where m = 1, 2, 3, ..., g

or in matrix form:

$$\begin{bmatrix} \sum_{j=0}^{kT-1} j^0 & \sum_{j=0}^{kT-1} j^1 & \ldots. & \sum_{j=0}^{kT-1} j^h \\ \sum_{j=0}^{2kT-1} j^0 & \sum_{j=0}^{2kT-1} j^1 & \ldots. & \sum_{j=0}^{2kT-1} j^h \\ \cdot & \cdot & \cdot & \cdot \\ \sum_{j=0}^{gkT-1} j^0 & \sum_{j=0}^{gkT-1} j^1 & \ldots. & \sum_{j=0}^{gkT-1} j^h \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_h \end{bmatrix} = \begin{bmatrix} 1-C_1 \\ 1-C_2 \\ \cdot \\ \cdot \\ 1-C_g \end{bmatrix} \tag{2.3-2}$$

or

$$J * A = Z \qquad (2.3\text{-}3)$$

where

$$J = \begin{bmatrix} \sum_{j=0}^{kT-1} j^o & \sum_{j=0}^{kT-1} j^1 & \cdots & \sum_{j=0}^{kT-1} j^h \\ \sum_{j=0}^{2kT-1} j^0 & \sum_{j=0}^{2kT-1} j^1 & \cdots & \sum_{j=0}^{2kT-1} j^h \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{j=0}^{gkT-1} j^0 & \sum_{j=0}^{gkT-1} j^1 & \cdots & \sum_{j=0}^{gkT-1} j^h \end{bmatrix}$$

$$Z = \begin{bmatrix} 1-C_1 \\ 1-C_2 \\ \cdot \\ \cdot \\ 1-C_g \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_h \end{bmatrix}$$

The solution A can be discussed in two different cases:

(i) If J is nonsingular, then

$$A = J^{-1} * Z \qquad (2.3\text{-}4)$$

(ii) If J is singular and full column rank, then

$$A = (J^T J)^{-1} J^T Z \qquad (2.3\text{-}5)$$

2.4 Sequential Refinement Schemes

To approximate the failure dynamics by a polynomial, the proper choice of the order h is, in general, quite difficult and depends upon physical considerations and the engineering experience. Once h is preselected, employing the technique developed in the previous section, the polynomial coefficients to best approximate the failure dynamics can be calculated. The accuracy of the failure dynamics depends greatly on the choice of the order and can be improved by considering more measurements. To find a new set of coefficients for a different combination of h and g, the entire calculation procedure must be repeated from the very beginning. This repitition is impractical in the on-line maintenance system. Therefore, sequential refinement schemes for obtaining new sets of coefficients without having to repeat the entire calculation will be developed for the case in which g and h are allowed to vary. It is assumed that the coefficients matrix A, found from the last section for an h order polynomial with g measurements, are solutions for eq. (2.3-2). Three typical cases will be considered in the succeeding paragraphs.

Case 1:

Suppose that an additional measurement C_{g+1} is taken at the next scheduled maintenance interval $(g+1)T$, then eq. (2.3-2) becomes:

$$\begin{bmatrix} \sum_{j=0}^{kT-1} j^0 & \sum_{j=0}^{kT-1} j^1 & \cdots & \sum_{j=0}^{kT-1} j^h \\ \sum_{j=0}^{2kT-1} j^0 & \sum_{j=0}^{2kT-1} j^1 & \cdots & \sum_{j=0}^{2kT-1} j^h \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{j=0}^{gkT-1} j^0 & \sum_{j=0}^{gkT-1} j^1 & \cdots & \sum_{j=0}^{gkT-1} j^h \\ \hline \sum_{j=0}^{(g+1)kT-1} j^0 & \sum_{j=0}^{(g+1)kT-1} j^1 & \cdots & \sum_{j=0}^{(g+1)kT-1} j^h \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_h \end{bmatrix} = \begin{bmatrix} 1-C_1 \\ 1-C_2 \\ \cdot \\ \cdot \\ 1-C_g \\ \hline 1-C_{g+1} \end{bmatrix} \qquad (2.4\text{-}1)$$

or

$$\begin{bmatrix} J \\ -- \\ \hat{J} \end{bmatrix} * \hat{A} = \begin{bmatrix} Z \\ -- \\ \hat{Z} \end{bmatrix} \qquad (2.4\text{-}2)$$

where J and Z were defined in eq. (2.3-3)

$$\hat{J} = \begin{bmatrix} \sum_{j=0}^{(g+1)kT-1} j^0 & \sum_{j=0}^{(g+1)kT-1} j^1 & \cdots & \sum_{j=0}^{(g+1)kT-1} j^h \end{bmatrix}$$

$$\hat{A} = [a_0 \; a_1 \; \ldots \; a_h]$$

$$\hat{Z} = 1-C_{g+1}$$

The matrix $\hat{A}$ can be solved as:

$$\hat{A} = (J^TJ + \hat{J}^T\hat{J})^{-1}(J^TZ + \hat{J}^T\hat{Z}) \qquad (2.4\text{-}3)$$

To avoid the calculation of a new matrix inverse as indicated in eq. (2.4-3) a sequential algorithm (24, 25) may be used to update $\hat{A}$. That is, assume

$$P = (J^TJ)^{-1}$$

is given, then from eq. (2.3-5)

$$A = PJ^TZ$$

and,

$$(J^TJ + \hat{J}^T\hat{J})^{-1} = P - P\hat{J}^T(\hat{J}P\hat{J}^T + I)^{-1}\hat{J}P \qquad (2.4\text{-}4)$$

Therefore,

$$\begin{aligned}\hat{A} &= P - P\hat{J}^T(\hat{J}P\hat{J}^T + I)^{-1}\hat{J}P(J^TZ + \hat{J}^T\hat{Z}) \qquad (2.4\text{-}5)\\ &= A + P\hat{J}^T\{I - (\hat{J}P\hat{J}^T + I)^{-1}\hat{J}P\hat{J}^T\}\ \hat{Z} - P\hat{J}^T(\hat{J}P\hat{J}^T + I)^{-1}\hat{J}A\end{aligned}$$

using the identity

$$I - (\hat{J}P\hat{J}^T + I)^{-1}\hat{J}P\hat{J}^T = (\hat{J}P\hat{J}^T + I)^{-1} \qquad (2.4\text{-}6)$$

Eq. (2.4-5) becomes

$$\hat{A} = A + P\hat{J}^T(\hat{J}P\hat{J}^T + I)^{-1}(\hat{Z} - \hat{J}A)\ . \qquad (2.4\text{-}7)$$

Notice that the manipulation of the matrix $(\hat{J}P\hat{J}^T + I)^{-1}$ is a simple scalar inversion.

Case 2:

Suppose an h order polynomial has been used and it is then determined that the error is too large. It would be more desireable to use a higher order, H, polynomial without having to repeat the entire calculation. Then, eq. (2.3-2) becomes:

$$\left[\begin{array}{cccc|cc} \sum_{j=0}^{kT-1} j^o & \sum_{j=0}^{kT-1} j^1 & \cdots & \sum_{j=0}^{kT-1} j^h & \cdots & \sum_{j=0}^{kT-1} j^H \\ \sum_{j=0}^{2kT-1} j^o & \sum_{j=0}^{2kT-1} j^1 & \cdots & \sum_{j=0}^{2kT-1} j^h & \cdots & \sum_{j=0}^{2kT-1} j^H \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_{j=0}^{gkT-1} j^o & \sum_{j=0}^{gkT-1} j^1 & \cdots & \sum_{j=0}^{gkT-1} j^h & \cdots & \sum_{j=0}^{gkT-1} j^H \end{array}\right] \left[\begin{array}{c} a_o \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_h \\ \hline \cdot \\ a_H \end{array}\right] = \left[\begin{array}{c} 1-C_1 \\ 1-C_2 \\ \cdot \\ \cdot \\ 1-C_g \end{array}\right] \tag{2.4-8}$$

or

$$\left[\begin{array}{c|c} J & \hat{J} \end{array}\right] \left[\begin{array}{c} \hat{A} \\ \hline \hat{B} \end{array}\right] = Z \tag{2.4-9}$$

where J and Z were defined in eq. (2.3-3),

$$\hat{J} = \left[\begin{array}{ccc} \sum_{j=0}^{kT-1} j^{h+1} & \cdots & \sum_{j=0}^{kT-1} j^H \\ \cdots & \cdots & \cdot \\ \sum_{j=0}^{gkT-1} j^{h+1} & \cdots & \sum_{j=0}^{gkT-1} j^H \end{array}\right]$$

$$\hat{A} = \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_h \end{bmatrix} \quad \text{and} \quad \hat{B} = \begin{bmatrix} a_{h+1} \\ a_{h+2} \\ \cdot \\ \cdot \\ a_H \end{bmatrix}$$

Assume A is the solution for the eq. (2.3-5), that is,

$$A = (J^T J)^{-1} J Z \tag{2.4-10}$$

The new solution matrices $\hat{A}$, $\hat{B}$ can be expressed as

$$\begin{aligned} \begin{bmatrix} \hat{A} \\ \\ \hat{B} \end{bmatrix} &= \left(\begin{bmatrix} J^T \\ \\ \hat{J}^T \end{bmatrix} [J \quad \hat{J}] \right)^{-1} \begin{bmatrix} J^T \\ \\ \hat{J}^T \end{bmatrix} Z \\ &= \begin{bmatrix} J^T J & J^T \hat{J} \\ \\ \hat{J}^T J & \hat{J}^T \hat{J} \end{bmatrix}^{-1} \begin{bmatrix} J^T \\ \\ \hat{J}^T \end{bmatrix} Z \end{aligned} \tag{2.4-11}$$

In order to improve the computational efficiency, an algorithm (24, 25) may be used to compute the matrix inverse. That is, assume

$$P = (J^T J)^{-1}$$
$$E = J^T \hat{J} \qquad (2.4\text{-}12)$$
$$F = \hat{J}^T \hat{J}$$

Then,

$$\begin{bmatrix} J^T J & J^T \hat{J} \\ \hat{J}^T J & \hat{J}^T \hat{J} \end{bmatrix}^{-1} = \begin{bmatrix} P^{-1} & E \\ E^T & F \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} P(I+ESE^T P) & -PES \\ -SE^T P & S \end{bmatrix} \qquad (2.4\text{-}13)$$

where,

$$S = (F - E^T PE)^{-1}$$
$$= (\hat{J}^T \hat{J} - \hat{J}^T JPJ^T \hat{J})^{-1} \qquad (2.4\text{-}14)$$

The new solution for $\hat{B}$ is found as follows using eq. (2.4-11) and eq. (2.4-13)

$$\hat{B} = -SE^T PJ^T Z + S\hat{J}^T Z$$
$$= S\hat{J}^T (Z - JPJ^T Z) \qquad (2.4\text{-}15)$$
$$= S\hat{J}^T (Z - JA)$$

The solution $\hat{A}$ is found in a similar way, i.e.

$$\hat{A} = P(I+ESE^TP)J^TZ - PES\hat{J}^TZ$$

$$= PJ^TZ + PESE^TPJ^TZ - PES\hat{J}^TZ$$

$$= A - PE(S\hat{J}^TZ - SE^TPJ^TZ) \tag{2.4-16}$$

$$= A - PJ^T\hat{J}\hat{B}$$

Case 3:

Suppose the order of the polynomial h is chosen to equal g, and whenever an additional measurement is taken at next maintenance interval, the order of the polynomial will be increased by 1. Then, eq. (2.3-4) becomes:

$$\left[\begin{array}{cccc|c} \sum_{j=0}^{kT-1} j^0 & \sum_{j=0}^{kT-1} j^1 & \cdots & \sum_{j=0}^{kT-1} j^g & \sum_{j=0}^{kT-1} j^{g+1} \\ \sum_{j=0}^{2kT-1} j^0 & \sum_{j=0}^{2kT-1} j^1 & \cdots & \sum_{j=0}^{2kT-1} j^g & \sum_{j=0}^{2kT-1} j^{g+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_{j=0}^{gkT-1} j^0 & \sum_{j=0}^{gkT-1} j^1 & \cdots & \sum_{j=0}^{gkT-1} j^g & \sum_{j=0}^{gkT-1} j^{g+1} \\ \hline \sum_{j=0}^{(g+1)kT-1} j^0 & \sum_{j=0}^{(g+1)kT-1} j^1 & \cdots & \sum_{j=0}^{(g+1)kT-1} j^g & \sum_{j=0}^{(g+1)kT-1} j^{g+1} \end{array}\right] \left[\begin{array}{c} a_0 \\ a_1 \\ \vdots \\ a_g \\ \hline a_{g+1} \end{array}\right] = \left[\begin{array}{c} 1-C_1 \\ 1-C_2 \\ \vdots \\ 1-C_g \\ \hline 1-C_{g+1} \end{array}\right]$$

or (2.4-17)

$$\left[\begin{array}{c|c} J & H \\ \hline \hat{J} & G \end{array}\right] \left[\begin{array}{c} \hat{A} \\ \hline \hat{B} \end{array}\right] = \left[\begin{array}{c} Z \\ \hline \hat{Z} \end{array}\right] \qquad (2.4\text{-}18)$$

where J, Z were defined in eq. (2.3-3), $\hat{J}$, $\hat{A}$, $\hat{Z}$ were defined in eq. (2.4-2) with h = g, and,

$$H = \begin{bmatrix} \sum_{j=0}^{kT-1} j^{g+1} \\ \sum_{j=0}^{2kT-1} j^{g+1} \\ \cdots \\ \sum_{j=0}^{gkT-1} j^{g+1} \end{bmatrix}$$

$$G = \sum_{j=0}^{(g+1)kT-1} j^{g+1} \quad \text{and} \quad \hat{B} = a_{g+1}$$

That is the matrices A and B can be expressed as

$$\begin{bmatrix} \hat{A} \\ \text{--} \\ \hat{B} \end{bmatrix} = \left[\begin{array}{c|c} J & H \\ \hline \hat{J} & G \end{array}\right]^{-1} \begin{bmatrix} Z \\ \text{--} \\ \hat{Z} \end{bmatrix} \tag{2.4-19}$$

A similar algorithm as used in case 2 can be employed (24, 25). That is,

$$\begin{bmatrix} J & H \\ \hat{J} & G \end{bmatrix}^{-1} = \begin{bmatrix} J^{-1}(I+HS\hat{J}J^{-1}) & -J^{-1}HS \\ -S\hat{J}J^{-1} & S \end{bmatrix} \tag{2.4-20}$$

where $S = (G - \hat{J}J^{-1}H)^{-1}$

From eq. (2.4-19) and eq. (2.4-20), the new solution A, B can be solved as:

$$\hat{B} = -S\hat{J}J^{-1}Z + S\hat{Z}$$

$$= S(\hat{Z} - \hat{J}A) \qquad (2.4\text{-}21)$$

$$\hat{A} = J^{-1}(I+HS\hat{J}J^{-1})Z - J^{-1}HS\hat{Z}$$

$$= J^{-1}Z + J^{-1}HS\hat{J}J^{-1}Z - J^{-1}HS\hat{Z}$$

$$= A - J^{-1}H(S\hat{Z} - S\hat{J}J^{-1}Z) \qquad (2.4\text{-}22)$$

$$= A - J^{-1}H\hat{B}$$

CHAPTER 3

FAULT PREDICTION

For the fault prediction algorithm it is assumed that the value of a component, drifting due to shock damages, is known at a fixed set of points in the "time domain". Using this data, one estimated the unknown failure dynamics for the parameter. This is then used to compute the component's lifetime in the "shock domain"; that is, the number of shocks required to cause a failure.

In the following paragraphs some formulas and properties of component's statistics will be developed. Using estimated lifetimes, some decision rules will be introduced to compute the optimal time at which to replace the component so as to minimize the cost function. Several terms are defined as follows:

(1) L : The component's estimated lifetime in the shock domain; i.e. the number of shocks required to cause a failure.

(2) T_r : The optimal time at which to replace the component so as to minimize the cost function.

(3) P_f : The probability that the failure occurs before the replacement time T_r.

(4) P_r : The probability that the component is replaced by a new component at replacement time T_r; i.e. the probability that the component's lifetime is longer than T_r.

(5) $f_L(t)$: The probability density function that the component receives the L^{th} shock at time t, where $0 < t \leq T_r$.

(6) $\tilde{T}_f$: The expected lifetime of components which failed before replacement.

(7) $\hat{T}$: The expected lifetime of components which either failed before T_r or were replaced at T_r.

(8) T^* : The expected lifetime of components; i. e. $T^* = T\Big]_{T_r \to \infty}$

Let $p_i(t)$ represent the density function of the Poisson distribution with parameter (kt) and $E_L(t)$ represent the corresponding distribution function; i.e.

$$p_i(t) = \frac{(kt)^i}{i!} e^{-kt} \qquad i = 0, 1, 2, ... \tag{3-1}$$

$$E_L(t) = \sum_{i=0}^{L-1} p_i(t) \tag{3-2}$$

Several properties can be stated as follows:

(prop 1) $P_r = E_L(T_r)$ (3-3)

$$P_r = \sum_{i=0}^{L-1} \frac{(kt)^i}{i!} e^{-kT_r}$$

$$= \sum_{i=0}^{L-1} p_i(T_r)$$

$$= E_L(T_r)$$

(prop 2) $P_f = 1 - E_L(T_r)$ (3-4)

$$P_f = \sum_{i=L}^{\infty} p_i(T_r)$$

$$= 1 - \sum_{i=0}^{L-1} p_i(T_r)$$

$$= 1 - E_L(T_r)$$

(Prop 3) $P_r + P_f = 1$ (3 -5)

(Prop 4) $$\int_0^{T_r} p_i(t)\, dt = \frac{1}{k}\{1 - E_{i+1}(T_r)\}$$ (3 -6)

$$\int_0^{T_r} p_i(t)\, dt = \int_0^{T_r} \frac{(kt)^i}{i\,!}\, e^{-kt}\, dt$$

$$= \frac{k^i}{i\,!}\int_0^{T_r} t^i\, e^{-kt}\, dt$$ (3 -7)

Using the identity eq. (3-8)

$$\int x^m\, e^{ax}\, dx = e^{ax} \sum_{r=0}^{m} (-1)^r \frac{m!\, x^{m-r}}{(m-r)\,!\, a^{r+1}}$$

(3 -8)

Eq. (3-7) becomes

$$\int_0^{T_r} p_i(t)\, dt = \frac{k^i}{i\,!}\, e^{-kt} \sum_{r=0}^{i} (-1)^r \frac{i\,!\, t^{i-r}}{(i-r)\,!\,(-k)^{r+1}}\Bigg]_0^{T_r}$$

$$= \frac{k^i}{i!}\left\{ e^{-k.0} \frac{i!}{k^{i+1}} - e^{-kT_r} \sum_{r=0}^{i} \frac{i!\, T_r^{\,i-r}}{(i-r)\,!\, k^{r+1}}\right.$$

$$= \frac{1}{k}\left\{ 1 - e^{-kT_r} \sum_{r=0}^{i} \frac{(kT_r)^{i-r}}{(i-r)\,!} \right\}$$

$$= \frac{1}{k} \{1 - e^{-kT_r} \sum_{j=0}^{i} \frac{(kT_r)^j}{j!}$$

$$= \frac{1}{k} \{1 - \sum_{j=0}^{i} p_j(T_r)\}$$

$$= \frac{1}{k} \{1 - E_{i+1}(T_r)\}$$

(Prop 5) $$f_L(t) = \frac{p_{L-1}(t)}{{}^1/_k \; (1-E_L(T_r))} \qquad (3\text{-}9)$$

where $0 < t \leq T_r$

If the interval $(0, T_r]$ is divided into N subintervals, it is seen that

$$\text{Prob } [(i-1)\Delta < t < i\,\Delta] = \frac{1}{N} \qquad (3\text{-}10)$$

where $\Delta = \frac{T_r}{N}$

Assume Δ is small enough such that the probability of more than one shock in the subinterval $((i-1)\Delta,\ i\Delta]$ is equal to 0. The probability of one shock in the subinterval $((i-1)\Delta,\ i\Delta]$ is equal to $k\Delta$, where k is the constant rate of Poisson shocks.

For a given t in the subinterval $((i-1)\Delta,\ i\Delta]$, the probability of the L^{th} shock in this subinterval is:

Prob {L^{th} shock occurs in

$((i-1)\Delta,\ i\Delta\ /\ ((i-1)\Delta < t \leq i\Delta)\}$

$= \text{Prob}\{(L-1) \text{ shocks occur in } (0,\ (i-1)\Delta]\}$

$* \text{ Prob }\{\text{one shock occurs in } ((i-1)\Delta,\ i\Delta]\}$

$$= \frac{[k(i-1)]^{L-1}}{(L-1)!} e^{-k(i-1)\Delta} * k\Delta$$

$$= p_{L-1}((i-1)\Delta) * k\Delta \qquad (3\text{-}11)$$

From Baylar theory,

Prob {have L^{th} in $((i-1)\Delta,\ i\Delta]\}$

$$= \frac{\text{Prob}\{(i-1)\Delta<t\leq i\Delta\}*}{\sum_{j=1}^{N} \text{Prob}\{(j-1)\Delta<\hat{t}\leq j\Delta\}*}$$

$$\frac{*\text{Prob}\{L^{th} \text{ shock occurs in } ((i-1)\Delta<t\leq i\Delta/(i-1)\Delta<t\leq i\Delta\}}{*\text{Prob}\{L^{th} \text{ shock occurs in } ((j-1)\Delta<\hat{t}\leq j\Delta/(j-1)\Delta<\hat{t}\leq j\Delta\}}$$

$$= \frac{\frac{1}{N}\, p_{L-1}((i-1)\Delta)\ \ k\Delta}{\sum_{j=1}^{N} \frac{1}{N}\, p_{L-1}((j-1)\Delta)\ \ k\Delta}$$

$$= \frac{p_{L-1}((i-1)\Delta)}{\sum_{j=1}^{N} p_{L-1}((j-1)\Delta)} \qquad (3\text{-}12)$$

Since Δ is very small, two approximations can be made as follows:

(i) $(i-1)\Delta \doteqdot t$

(ii) Left hand side of eq. (3-12) is approached to

$$f_L(t) * \Delta$$

That is, eq. (3-12) can be rewritten as

$$f_L(t) = \frac{p_{L-1}(t)}{\sum_{j=1}^{N} p_{L-1}((j-1)\Delta)\Delta}$$

If $\Delta \to 0$, by definition of the integral the summation becomes an integral, such that

$$f_L(t) = \frac{p_{L-1}(t)}{\int_0^{T_r} p_{L-1}(\hat{t})\, d\hat{t}} \qquad (3\text{-}14)$$

From (Prop 4), eq. (3-14) becomes

$$f_L(t) = \frac{p_{L-1}(t)}{\frac{1}{k}\{1-E_L(T_r)\}}$$

(Prop 6) $$\tilde{T}_f = \frac{L}{K} * \frac{1 - E_{L+1}(T_r)}{1 - E_L(T_r)}$$

Since $\tilde{T}_f$ is the expected lifetime of the components which failed before replacement,

$$\tilde{T}_f = \int_0^{T_r} t\, f_L(t)\, dt$$

$$= \int_0^{T_r} \frac{t\, p_{L-1}(t)}{\frac{1}{k}\{1-E_L(T_r)\}}\, dt$$

$$= \frac{\int_0^{T_r} t * \frac{(kt)^{L-1}}{(L-1)!} e^{-kt}\, dt}{\frac{1}{k}\{1 - E_L(T_r)\}}$$

$$= \frac{\frac{L}{k}\int_0^{T_r} \frac{(kt)^L}{L!} e^{-kt}\, dt}{\frac{1}{k}\{1-E_L(T_r)\}}$$

$$= \frac{L * \int_0^{T_r} p_L(t)\, dt}{\{1 - E_L(T_r)\}} \qquad (3\text{-}16)$$

From (Prop 4), eq. (3-16) becomes

$$\tilde{T}_f = \frac{L}{k} * \frac{\{1 - E_{L+1}(T_r)\}}{\{1 - E_L(T_r)\}}$$

(Prop 7) $$\hat{T} = \frac{L}{k}\{1 - E_{L+1}(T_r)\} + T_r E_L(T_r) \qquad (3\text{-}17)$$

$$\hat{T} = P_f \tilde{T}_f + P_r T_r$$

$$= \{1 - E_L(T_r)\} * \frac{L}{k} * \frac{1 - E_{L+1}(T_r)}{1 - E_L(T_r)} + T_r E_L(T_r)$$

$$= \frac{L}{k}\{1 - E_{L+1}(T_r)\} + T_r E_L(T_r)$$

(Prop 8) $$T^* = \frac{L}{k} \qquad (3\text{-}18)$$

$$E_i(T_r)\Big]_{T_r=\infty} = \sum_{j=0}^{i-1} \frac{(kt)^j}{j!} e^{-kt}\Big]_{t=\infty} \div 0$$

i. e. The probability of having a finite number of shocks in an infinite time period is zero.

$$T^* = \hat{T}\Big]_{T_r=\infty} = \frac{L}{k}\{1 - E_{L+1}(T_r)\} + T_r E_L(T_r)\Big]_{T_r=\infty}$$

$$= \frac{L}{k}$$

(Prop 9)

(1) $$\frac{d(P_f)}{d(kT_r)} = p_{L-1}(T_r) \qquad (3\text{-}19)$$

(2) $$\frac{d(P_r)}{d(kT_r)} = -p_{L-1}(T_r) \tag{3-20}$$

$$P_r = E_L(T_r)$$

$$= \sum_{i=0}^{L-1} \frac{(kT_r)^i}{i!} e^{-kT_r}$$

$$= e^{-kT_r} + \sum_{i=1}^{L-1} \frac{(kT_r)^i}{i!} e^{-kT_r}$$

$$\therefore \frac{d(P_r)}{d(kT_r)} = -e^{-kT_r} + \sum_{i=1}^{L-1} \left\{\frac{i(kT_r)^{i-1}}{i!} - \frac{(kT_r)^i}{i!}\right\} e^{-kT_r}$$

$$= \sum_{i=1}^{L-1} \frac{(kT_r)^{i-1}}{(i-1)!} e^{-kT_r} - \sum_{i=0}^{L-1} \frac{(kT_r)^i}{i!} e^{-kT_r}$$

$$= E_{L-1} - E_L$$

$$= -p_{L-1}(T_r)$$

$$\because P_f = 1 - P_r$$

$$\therefore \frac{d(P_f)}{d(kT_r)} = p_{L-1}(T_r)$$

(Prop 10) $$\frac{d(kT_f)}{d(kT_r)} = L \frac{\{1-E_L(T_r)\}p_L(T_r) - \{1-E_{L+1}(T_r)\}p_{L-1}(T_r)}{[1-E_L(T_r)]^2} \tag{3-21}$$

$$k\tilde{T}_f = L * \frac{1 - E_{L+1}(T_r)}{1 - E_L(T_r)}$$

$$\frac{d(kT_f)}{d(kT_r)} = L * \frac{\{1-E_L(T_r)\}\ p_L(T_r) - \{1-E_{L+1}(T_r)\}\ p_{L-1}(T_r)}{\{1 - E_L(T_r)\}^2}$$

(Prop 11) $$\frac{d(k\hat{T})}{d(kT_r)} = E_L(T_r) \qquad (3\text{-}22)$$

$$\hat{T} = \frac{L}{k}\{1 - E_{L+1}(T_r)\} + T_r E_L(T_r)$$

$$k\hat{T} = L\{1 - E_{L+1}(T_r)\} + kT_r E_L(T_r)$$

$$\frac{d(k\hat{T})}{d(kT_r)} = L\{p_L(T_r)\} + kT_r(-p_{L-1}(T_r)) + E_L(T_r)$$

$$= L\, p_L(T_r) - kT_r p_{L-1}(T_r) + E_L(T_r)$$

Since,

$$L\, p_L(T_r) = L * \frac{(kT_r)^L}{L!} e^{-kT_r}$$

$$= (kT_r) * \frac{(kT_r)^{L-1}}{(L-1)!} e^{-kT_r}$$

$$= kT_r\, p_{L-1}(T_r)$$

$$\therefore \frac{d(k\hat{T})}{d(kT_r)} = E_L(T_r)$$

Once the estimated failure time and statistics of the component under study have been computed it remains to make a replacement decision. This should be based on the probability of the component's failure "P_f", the cost of replacing the component "C_R", and the cost of an on-line failure "C_f". That is, the cost function to be optimized can be expressed as either

$$\text{Cost} = \frac{1}{\hat{T}} (C_R + C_f P_f) \tag{3-23}$$

or

$$\text{Cost} = C_f P_f + C_R P_r \tag{3-24}$$

One possible replacement criterion is based on the cost of an on-line failure and average lifetime wastage due to replacing the component before its actual failure.

$$\text{Cost} = C_f P_f + C_W (kT^* - k\hat{T}) \tag{3-25}$$

where C_f and C_W are two weight constants. The first term of eq. (3-25) represents the cost of an on-line failure and the second term represents the lifetime wastage. The replacement time T_r which simultaneously minimizes the cost of an on-line failure and lifetime wastage should satisfy eq.(3-26) or eq.(3-27).

$$0 = C_f \, P_{L-1}(T_r) - C_W \, E_L(T_r) \tag{3-26}$$

therefore,

$$\left(\frac{C_f}{C_W} - 1\right) * \frac{(kT_r)^{L-1}}{(L-1)!} = \sum_{i=0}^{L-2} \frac{(kT_r)^i}{i!} \qquad (3\text{-}27)$$

Table (3-1) gives the replacement time kT_r with different weight constants (C_f/C_W) and different estimated lifetimes. Fig. (3-1) shows T_r curves being a function of L with different (C_f/C_W).

Table (3-1)

The replacement time (kT_r) for different (C_f/C_W) and estimated lifetimes (L)

C_f/C_W \ L	3	4	5	6	7	8	9	10	15	20
2.0	2.73	4.59	6.5	8.44	10.39	12.35	14.3	16.29	26.2	36.2
3.0	1.62	2.89	4.24	5.62	7.03	8.46	9.89	11.34	18.65	26.1
4.0	1.22	2.27	3.4	4.58	5.79	7.02	8.26	9.52	15.9	22.4
5.0	1.0	1.93	2.95	4.01	5.11	6.23	7.37	8.52	14.4	20.4
6.0	0.86	1.71	2.65	3.64	4.67	5.72	6.79	7.87	13.4	19.2
7.0	0.77	1.56	2.44	3.37	4.35	5.35	6.37	7.41	12.8	18.3
8.0	0.70	1.44	2.28	3.17	4.11	5.07	6.06	7.06	12.2	17.6
9.0	0.64	1.35	2.15	3.01	3.92	4.85	5.8	6.78	11.8	17.
10.0	0.60	1.27	2.05	2.88	3.76	4.67	5.6	6.55	11.5	16.8
15.0	0.46	1.03	1.71	2.46	3.25	4.08	4.93	5.8	10.4	15.1
20.0	0.38	0.9	1.52	2.22	2.96	3.74	4.54	5.37	9.7	14.3

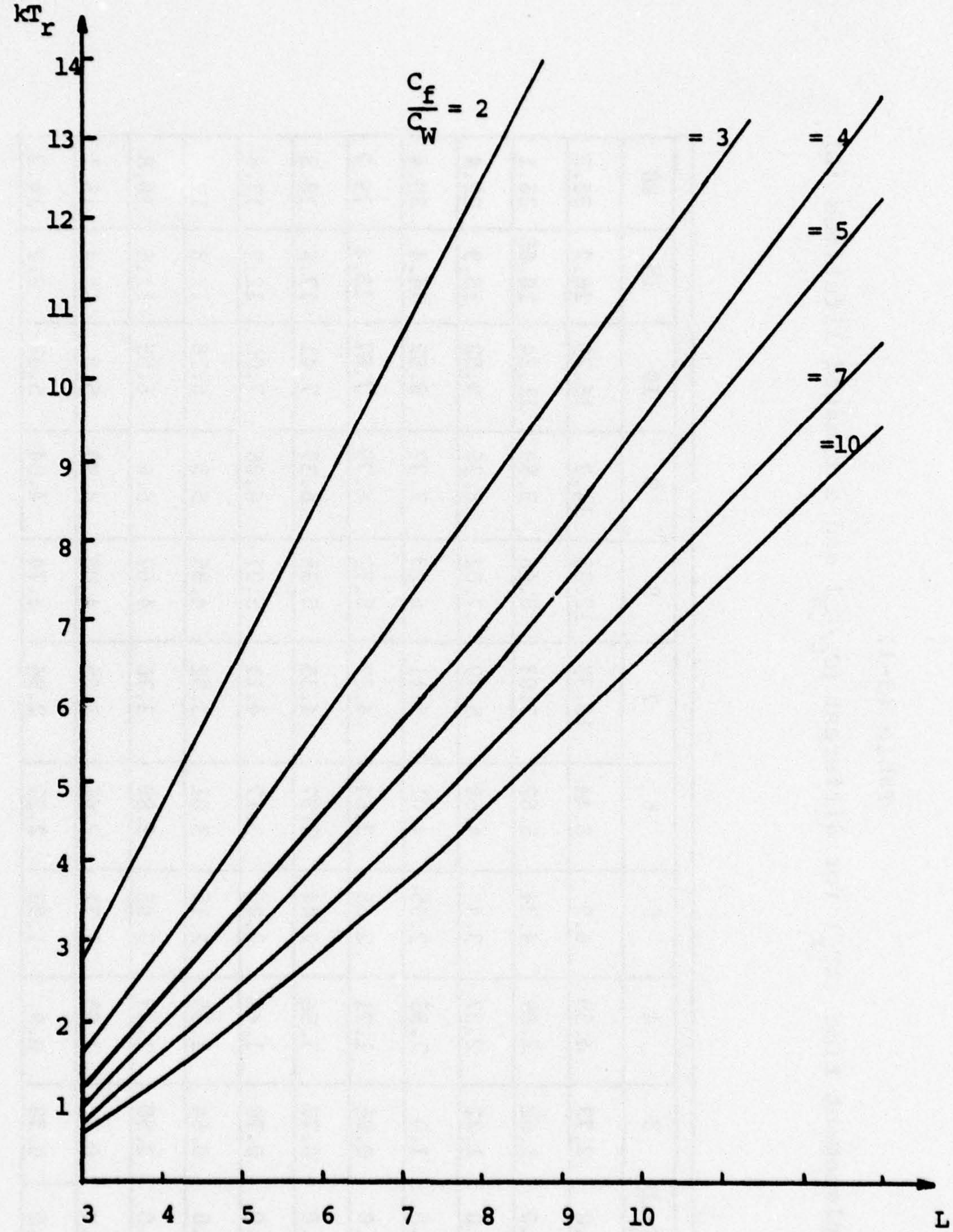

Fig. (3-1)

Replacement time (kT_r) v.s. Lifetime L with different weight constant

CHAPTER 4

SIMULATIONS

4.1 Poisson Shocks Generator

To generate a series of Poisson Shocks for the simulation of an on-line maintenance process by computer, uniformly distributed random numbers are needed. A linear congruential method (26, 27) can be used to generate numbers uniformly distributed on the interval (0, 1) by computer. That is, if the following relation exists between a set of numbers $\{x_i\}$

$$x_{n+1} = (x_n * (2^s+1) + b)_{\bmod m} \qquad (4.1\text{-}1)$$

where $s = Q/2, \quad m = 2^Q$

Q = the number of shift register bits in the computer

and b = any odd number

then, the sequence $\{x_i\}$ is of period m and $0 \leq x_i \leq m-1$. If each x_i is adjusted to the range (0, 1) by dividing by m; i.e., $u_i = x_i/m$, then u_i is a sequence of uniformly distributed random numbers between 0 and 1. That is,

$$p_U(u) = 1 \qquad (4.1\text{-}2)$$

where $0 \leq u < 1$

In order to convert the uniform distribution U to a Poisson distribution N, assume:

$$y = -\frac{1}{k} \ln (1-u) \tag{4.1-3}$$

or

$$u = 1 - e^{-ky} \tag{4.1-4}$$

where $y \geq 0$

and k is a positive constant.

By the change of variable method (28, 29), the probability density function, $p_Y(y)$, becomes

$$\begin{aligned} p_Y(y) &= p_U(u) * \left| \frac{du}{dy} \right| \\ &= 1 * ke^{-ky} \end{aligned} \tag{4.1-5}$$

Let y represent the time interval between shocks, i.e., the times of successive shocks are y_1, y_2, ... etc. Then, the distribution of the number of shocks, n, in a period T can be solved as follows:

$$\begin{aligned} \text{Prob}(N=n) &= \text{Prob}(y_1+y_2+\ldots+y_{n+1} \geq T) \\ &\quad -\text{Prob}(y_1+y_2+\ldots+Y_n \geq T) \end{aligned} \tag{4.1-6}$$

From eq. (4.1-5) it can be seen than Y is distributed as $\frac{1}{2}*\frac{1}{k}$ (χ^2 with 2 degrees of freedom) and $\sum_{j=1}^{m} y_j$ is distributed as $\frac{1}{2}*\frac{1}{k}$(χ^2 with 2m degrees of freeedom). Therefore, (30) eq. (4.1-6) becomes

$$\text{Prob}(N=n) = \text{Prob}(\chi^2_{2(n+1)} \geq 2kT) - \text{Prob}(\chi^2_{2n} \geq 2kT)$$

$$= \sum_{j=0}^{n} e^{-kT} \frac{(kT)^j}{j!} - \sum_{j=0}^{n-1} e^{-kT} \frac{(kT)^j}{j!}$$

$$= e^{-kT} * \frac{(kT)^n}{n!} \tag{4.1-7}$$

Hence N has a Poisson Distribution with parameter (kT). Therefore, shocks that occur according to this Y distribution are called Poisson Shocks.

4.2 Simulation of a Perfect System

The computer simulation of a perfect (no measurement errors, and identical drifting parameters) on-line maintenance system can be performed using the following steps:

Step 1 Define Constants

(1) T: maintenance interval

(2) k: time constant for Poisson Shock i.e. average number of shocks per unit time.

(3) x_0: seed for uniform distributed random number generator

(4) C_f/C_W: the ratio of two weight factors

Step 2 Generate Poisson Shock

(1) Generate Poisson Shock $y_1, y_2, y_3 \ldots$

(2) Calculate number of shocks received in each maintenance interval $n_1, n_2, n_3 \ldots$

Step 3 Define Values of Component Parameter

(1) $C(0) = 1$

(2) $C(i) = a_i$ where i is an integer and $0 < i < L$ and $0 < a_i \leq 1$

(3) $C(i) = 0$ where i is an integer and $i \geq L$ -- L is the lifetime of the component in the "Shock Domain".

Step 4 <u>Observe Component Performance at Next Maintenance Time mT</u>

(1) At maintenance time = mT, the total number of shocks received are $N = \sum_{i=1}^{m} n_i$, therefore the performance data

$C_m = C(N)$

(2) Case 1: If $C_m = 0$, then an on-line failure occurs.

GO TO STEP 9

Case 2: If $C_m > 0$, then the component performs properly.

GO TO STEP 5

Step 5 <u>Calculate Failure Dynamics</u>

The new set of constants $\{a_i\}$ of the failure dynamics can be calculated from eq. (2.4-23) and eq. (2.4-24)

Step 6 <u>Solve Component's Lifetime</u>

The lifetime of the component is then solved from eq. (2.1-5).

Step 7 <u>Estimate Component's Replacement Time</u>

The replacement time, T_r, can be predicted from eq. (3-27).

Step 8 <u>Make a Decision</u>

Case 1 : If $T_r < (m+1)T$, then the component will not be replaced.

GO TO STEP 4

Case 2: If $T_r \geq (m+1)T$, then the component will be replaced.

GO TO STEP 9

Step 9 Replace component

Replace the component and reset the count m to zero

GO TO STEP 4

Example:

The computer simulation of a on-line periodic maintenance system is performed for 600 maintenance intervals. Assume the system is subjected to a Poisson Shock with constant k=0.1 shocks/hour and each interval T is 20 hours. The normalized values of a drifting parameter due to shocks damage are as followings:

C(0) = 1.0	C(14) = 0.9
C(1) = 0.998	C(15) = 0.89
C(2) = 0.995	C(16) = 0.88
C(3) = 0.989	C(17) = 0.87
C(4) = 0.982	C(18) = 0.85
C(5) = 0.975	C(19) = 0.83
C(6) = 0.968	C(20) = 0.8
C(7) = 0.96	C(21) = 0.75
C(8) = 0.952	C(22) = 0.68
C(9) = 0.944	C(23) = 0.6
C(10)= 0.936	C(24) = 0.5
C(11)= 0.927	C(25) = 0.35
C(12)= 0.918	C(26) = 0.2
C(13)= 0.909	C(27) = 0.05

and, C(N) = 0 if N $\geq$28

Table (4.2-1) gives total number of replacements and failures within 600 maintenance intervals with different values of

C_f/C_W. Table (4.2-2) shows the total number of replacements and failures with different replacement times.

Since the cost function is

$$\text{Cost} = C_f{*}\ P_f \ + \ C_W{*}\ (kT^* - k\hat{T})$$

the overall cost can be expressed as

$$\text{Cost} = \frac{C_f}{C_W} * \text{(No. of failures)}$$

$$+\ 0.1 * (280*(\text{No. of components used}) - 12000)$$

The overall cost with different methods and different values of C_f/C_W are given in Table (4.2-3).

Table (4.2-1)

Total replacements and failures within 600 maintenance intervals for different C_f/C_W

C_f/C_W	No. of replacement	No. of failure
50	48	7
75	56	1
100	52	2
150	54	2
200	54	2

Table (4.2-2)

Total replacements and failures within 600 maintenance intervals for methods of constant time replacement

Constant replacement time	No. of replacement	No. of failure
every 6 intervals	100	0
every 7 intervals	85	0
every 8 intervals	75	0
every 9 intervals	65	1
every 10 intervals	59	1
every 11 intervals	48	6
every 12 intervals	39	11

Table (4.2-3)

Overall cost with different methods and different C_f/C_W

Methods \ Cost C_f/C_W	50	75	100	150	200
every 6 intervals	1600	1600	1600	1600	1600
every 7 intervals	1096	1096	1096	1096	1096
every 8 intervals	900	900	900	900	900
every 9 intervals	698	723	748	798	848
every 10 intervals	530	555	580	630	680
every 11 intervals	612	762	912	1212	1512
every 12 intervals	750	1025	1300	1850	2400
the algorithm	690	471	512	668	763

4.3 Simulation of an Imperfect System

The computer simulation of an imperfect on-line maintenance system can be performed by modifying the previous algorithm. Instead of using the same set of C(i) for every component, a noise is added into C(i) such that the lifetime of each component may not be the same but the average lifetime is still unchanged.

The total number of replacements and failures with $C_f/C_W = 100$ in 600 maintenance intervals are solved by using different methods. The results with different noise levels are showed in Table (4.3-1).

Table (4.3-2) gives the overall cost with different methods and different noise levels.

Table (4.3-1)

Overall cost with different noise levels

noise level / method	20%		30%		40%		60%	
	No. of replace	No. of fail	No. of replace	No. of fail	No. of replace	No. of fail	No. of replace	No. of fail
every 6 intervals	100	0	100	0	100	0	94	6
every 7 intervals	85	0	85	0	84	1	78	8
every 8 intervals	75	0	72	3	71	4	64	12
every 9 intervals	64	2	63	3	60	7	52	17
every 10 intervals	56	4	51	9	45	15	45	18
every 11 intervals	45	10	45	10	45	10	39	20
every 12 intervals	36	15	35	16	36	17	31	23
the algorithm	56	3	55	5	55	5	50	11

Table (4.3-2)

Overall cost for different methods at different noise levels

cost / noise level / method	0 %	20 %	30 %	40 %	60 %
every 6 intervals	1600	1600	1600	1600	2200
every 7 intervals	1096	1096	1096	1280	2008
every 8 intervals	900	900	1200	1300	2128
every 9 intervals	748	848	948	1376	2432
every 10 intervals	580	880	1380	1980	2364
every 11 intervals	912	1340	1340	1340	2452
every 12 intervals	1300	1728	1828	1984	2612
the algorithm	512	752	980	752	1608

CHAPTER 5

CONCLUSION

In the preceeding chapters we have described a fault prediction algorithm for on-line maintenance systems. This algorithm essentially can be separated into three major steps: 1) applying a curve fitting technique to solve the failure dynamics by using past performance data, 2) predicting the lifetime of the system in the shock domain from the failure dynamics, 3) estimating the replacement time which simultaneously minimizes the cost of an on-line failure and lifetime wastage. In addition, sequential refinement schemes have been developed to solve the problem of inverting a potentially high dimensional matrix. Thus without having to repeat the entire calculation, the new sets of failure dynamics can be obtained recursively based on the old estimates and new data.

The algorithm has been tested in a variety of situations such as: perfect and imperfect system, different levels of noise, different sets of Poisson shocks. The results have been found to be surprisingly effective in predicting failures with relatively little wastage of lifetime and on-line failure cost.

Finally, similar algorithms for the replacement criterion based on the cost functions of eq. (4.1-23) and eq. (4.1-24) have been studied using the properties introduced in chapter 3. These algorithms yielded very good results.

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APPENDIX

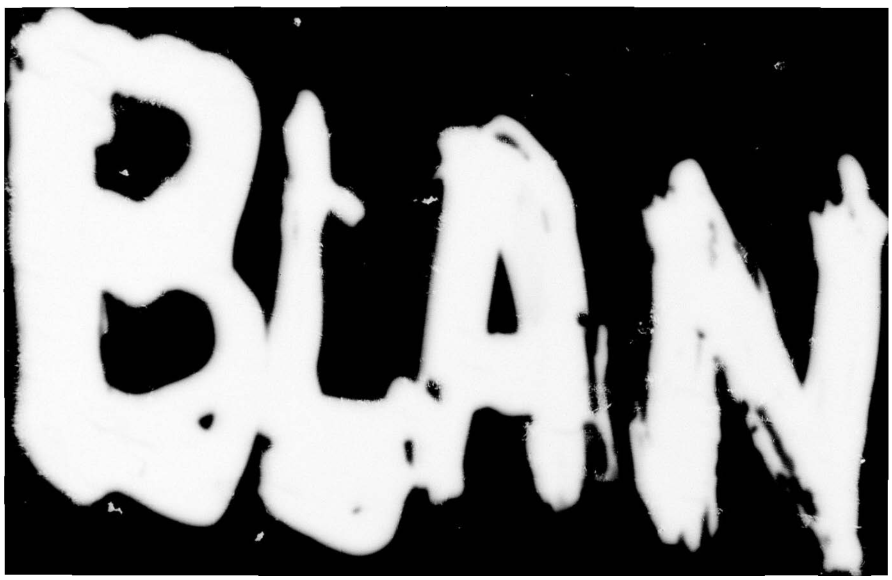
FORTRAN PROGRAMS FOR THE SIMULATION SYSTEM

```
// FOR MAX    01 JUN 77 23.409 HRS
*LIST ALL
*IOCS(CARD,DISK,1443 PRINTER)
*ONEWORD INTEGERS
*NONPROCESS PROGRAM
      SUBROUTINE MAX(ICODE,AM,BM,CM,M,N,K)
C.
C.  THIS IS THE SUBROUTINE FOR MATRIX OPERATION
C.  AM(M,N)     BM(N,K)      CM(M,K)
C.  ICODE=1     CM=AM+BM
C.  ICODE=2     CM=AM-BM
C.  ICODE=3     CM=AM*BM
C.
       DIMENSION AM(20,20),BM(20,20),CM(20,20)
       IF(ICODE-2 ) 100,200,300
100    CONTINUE
       DO 10 I=1,N
       DO 10 J=1,N
       CM(I,J) = AM(I,J) + BM(I,J)
10     CONTINUE
       GO TO 400
200    CONTINUE
       DO 20 I=1,N
       DO 20 J=1,N
       CM(I,J) = AM(I,J) - BM(I,J)
20     CONTINUE
       GO TO 400
300    CONTINUE
       DO 30 I=1,M
       DO 30 J=1,K
       CM(I,J) = 0.0
       DO 5 L=1,N
       CM(I,J) = CM(I,J) + AM(I,L)*BM(L,J)
5      CONTINUE
30     CONTINUE
400    CONTINUE
       RETURN
       END
// FOR RNGT    01 JUN 77 23.412 HRS
*IOCS(CARD,DISK,1443 PRINTER)
*LIST ALL
*NONPROCESS PROGRAM
*ONEWORD INTEGERS
       INTEGER CR,PR
       DIMENSION AM(20,20),BM(20,20),CM(20,20)
       DIMENSION TSHUK(3000),CH(40)
       DIMENSION E(1,20),H(20,1),A(20,1)
       DIMENSION    C(20,1),FIN(20,20)
       DIMENSION DUM(20,20),DUM1(20,20)
       DIMENSION DUM2(20,20),FINEW(20,20)
```

```
      DIMENSION          NSK(600),IDESN(600), OB(20)
      EQUIVALENCE(FIN(1,1),TSHUK(1) )
      EQUIVALENCE(FINEW(1,1),TSHUK(401),DUM2(1,1))
      EQUIVALENCE    (  DUM1(1,1),TSHUK(801) )
      EQUIVALENCE(DUM(1,1),TSHUK(1201) )
      EQUIVALENCE(A(1,1),TSHUK(1601))
      EQUIVALENCE(H(1,1),TSHUK(1621))
      EQUIVALENCE   ( E(1,1),TSHUK(1641))
      EQUIVALENCE(C(1,1),TSHUK(1661) )
      EQUIVALENCE(OB( 1),TSHUK(1681))
      EQUIVALENCE(CM(1,1),TSHUK(1701))
      EQUIVALENCE(AM(1,1),TSHUK(2101))
      EQUIVALENCE(BM(1,1),TSHUK(2501))
      EQUIVALENCE(CH(1  ),TSHUK(2901))
      CR=5
      PR=6
C.
C.  THIS SECTION IS TO GENERATED POISSON SHOCK
C.
C.  SEED   THE SEED FOR THE R. N. GENERATOR
      READ(CR,9101)  SEED
9101  FORMAT(F 5.0)
C.  NRND   NUM OF RANDOM NUMBER TO BE GENERATED
      READ(CR,9101) NRND
9101  FORMAT(I4)
C.
C.  INPUT THE OBSERVATION PERIOD TPD
C.
      READ(CR,901) TPD
901   FORMAT(F15.5)
C.  TCNST   TIME CONSTANT
      READ (CR,9101) TCNST
      CNST=TPD/TCNST
      WRITE(PR,9106) SEED
9106  FORMAT(1H ,'THE SEED FOR THE RANDOM NO. IS ' ,F5.0  )
      WRITE(PR,9107) NRND
9107  FORMAT(1H ,'THE NO. OF POISSON SHOCK TO BE GENERATED IS ', I5)
      WRITE(PR,9108) TCNST
9108  FORMAT(1H ,'AVE. NO. OF SHOCK PER OBSERVATION PERIOD IS ' ,F5.0)
C.
C.  X(N+1)=(X(N)*AN +BN) MOD CN
C.  X(N)  IS R.N.   N(0,16383)
C.
      DO 9115 INRND=1,NRND
      AN=129.0
      BN=111.0
      CN=16384.0
      S1=SEED*AN +BN
      IS=S1/CN
      S2=S1-IS*CN
```

```
      SEED=S2
C.
C.  SEER  NORMOLIZED R. N.   N(0,1)
C.  TSHUK IS POISSON SHUCK
C.
      SEER=SEED/CN
      TSHUK(INRND) = (- CNST)*ALOG(1-SEER)
9115  CONTINUE
      JC= (NRND-1) /10 +1
      WRITE(PR,9130)
9130  FORMAT(1H , "THE SET OF POISSON SHOCK ARE ')
      DO 9125 J=1, JC
      LL=(J-1)*10+1
      LH=J*10
      WRITE(PR,9135) (TSHUK(I), I=LL,LH)
9135  FORMAT (1H ,10(F10.5,1X))
9125  CONTINUE
C.
C.  OUTPUT THE OBSERVATION PERIOD TPD
C.
      WRITE(PR,910) TPD
910   FORMAT(1H ,"THE OBSERVATION PERIOD IS ' ,F15.5)
      I=0
      J=1
      LL=0
925   SUM=0.0
915   I=I+1
      IF(I-NRND) 918,918,930
918   SUM=TSHUK(I)+SUM
      IF(SUM-TPD) 915,920,920
920   IF(J-600) 924,924,922
922   WRITE(PR,923)
923   FORMAT(1H ,'THE DIMENSION OF NSK ARRAY IS TO SMALL')
      GO TO 930
924'  NSK(J) = I-LL-1
      LL=I-1
      TSHUK(I) = SUM-TPD
      I=I-1
      J=J+1
      GO TO 925
930   MAXPT=J-1
      DO  940 I=J,600
940   NSK(I)=0
      WRITE(PR,945)
 945  FORMAT(1H ,'THE NO. OF SHOCK AT EACH OBSERVATION PERIOD ARE ')
      JC= (MAXPT-1) /10  +1
      DO 950 J=1,JC
      LL=(J-1)*10 +1
      LH=J*10
      WRITE(PR,946) (NSK(I),I=LL,LH)
```

```
946    FORMAT(1H ,10 (I5,2X))
950    CONTINUE
       ICNP=5
       DO 100 IC=1,12
       ICNP=ICNP +1
       WRITE(PR,101) ICNP
101    FORMAT(/////,' PERIOD ',I4)
       ICNPT=0
       ISUM=0
       DO 105 I=1,MAXPT
       ICNPT=ICNPT +1
       ISUM=ISUM+NSK(I)
       IF(ISUM-28) 110,120,120
110    IF(ICNPT-ICNP) 105,111,111
  111  WRITE(PR,115)I
115    FORMAT(' AT TEST POINT ', I 5,'REPLACE')
       ICNPT=0
       ISUM=0
       GO TO 105
120    WRITE(PR,125) I
125    FORMAT(' AT TEST POINT', I 5,' FAIL')
       ISUM=0
       ICNPT=0
105    CONTINUE
100    CONTINUE
C.
C.  INPUT THE CHARACTERISTIC OF THE DEVICE
C.
C.  C(0) = 1
C.  C(N+1) ALWAYS LESS THAN C(N)
C.   4 DATA CARDS WITH 10(F6.5 1X)
       WRITE(PR,9315)
9315   FORMAT(1H ,"THE CHARACTERISTIC OF THE DEVICE ARE ')
       READ(CR,9310) I
9310   FORMAT(78X,A1)
       DO 9300 I=1,4
       LL=(I-1)*10 +1
       LH=I*10
       READ(CR,9350)(CH(J),J=LL,LH)
9350   FORMAT(  10(F6.5,1X))
       WRITE(PR,9360)  (CH(J),J=LL,LH)
9360   FORMAT(1H ,10(F7.5,1X))
9300   CONTINUE
C.
C.  INPUT THE RATIO OF   CF/CW
C.  CODE FOR IDESN ARE
C.    IDESN(IDE) = 1     FAIL
C.    IDESN(IDE) = 2     CONTINUE
C.    IDESN(IDE) = 3     REPLACE
```


BLAN

```
C.
      MT=TCNST
      READ(CR,3002) CFW
3002  FORMAT(F15.5)
      WRITE(PR,3003) CFW
3003  FORMAT(1H ,'THE RATIO CF/CW IS '   ,F15.5)
      MT=TCNST
      IDE=0
3005  ICT=0
3006  IDE=IDE+1
      IF(IDE-MAXPT) 3010,3010,4000
3010  ICT=ICT+1
      IF(ICT-1) 3015,3015,3115
3015   ITSK= NSK(IDE)
      IF(CH(ITSK)) 3020,3020,3025
C.
C.  FIRST TIME TEST AND FAIL
C.
3020  WRITE(PR,3022)IDE,ITSK
3022  FORMAT(1H ,'AT TEST POINT',I5,3X,
     1'THE DEVICE TOTAL RECEIVED',I5,2X,
     2'SHOCKS AND FAIL TO OPERATE')
      IDESN(IDE)=1
      GO TO 3005
C.
C.  FIRST TIME BUT NOT FAIL
C.
3025  FIN(1,1)=1.0/MT
      OB(ICT) = CH(ITSK)
      C(1,1)  =  1.0 -OB(ICT)
      NORED=1
      A(1,1)=FIN(1,1)*C(1,1)
      GO TO 502
C.
C.
C.  THIS IS THE ROUTINE TO SOLVE THE PARAMETERS
C  NORED   THE ORDER OF THE COEFFICIENTS
C.  FI   THE INVERSE OF THE MATRIX F
C.  F  X A = C
C.  FH * A = C
C.  EG * B = D
C.  H  (NORED , 1)
C.  E  (1 , NORED)
C.  G , D IS CONSTANT
3115  CONTINUE
      ITSK=ITSK + NSK(IDE)
      IF (CH(ITSK)) 3020,3020,1000
1000  CONTINUE
C.
C.  CALCULATE H  C
```

```
C.      NOREl=NORED+1
        SUM=0.0
        DO 1010 I=1,NORED
        LML=(I-1) * MT
        IF(LML) 1002,1002,1003
1002    LML=1
1003    LMH= I*MT - 1
        DO 1005 J= LML , LMH
        SUM = SUM + J ** NOREl
1005    CONTINUE
        H(I,1)=SUM
        C(I,1)=1.0- OB(I)
1010    CONTINUE
C.
C.  CALCULATE G
C.
        LML= LMH+1
        LMH= LML+ MT
        DO 1110 J = LML ,LMH
        SUM= SUM + J ** NOREl
1110    CONTINUE
        G=SUM
C.
C.  CALCULATE E
C.
        DO  1111  I=2,NORED
1111    E(1,I) = 0.0
        E(1,1) = (NOREl*MT)
        LMH= (NOREl  ) * MT - 1
        DO 1120  J= 2,NORED
        DO 1115  I= 1, LMH
        E(1, J) =  E(1,J) + I ** J
1115    CONTINUE
1120    CONTINUE
        OB(NOREl)  =  CH(ITSK)
        D=1.0-OB(NOREl)
        CALL MAX (3,E,FIN,DUM,1,NORED,NORED)
        CALL MAX (3,DUM,H,DUM1,1,NORED,1)
        S=1.0/(G-DUM1(1,1))
        FINEW(NOREl,NOREl) =S
        DO 1500 J=1,NORED
        FINEW(NOREl,J) = -S *DUM(1,J)
1500    CONTINUE
        DO 1510 J=1,NORED
        DUM(1,J) = DUM(1,J) *S
1510    CONTINUE
        CALL MAX ( 3,FIN,H,DUM1,NORED,NORED,1)
        CALL MAX(3,DUM1,DUM,DUM2,NORED,1,NORED)
        CALL MAX (1,FIN,DUM2,FINEW,NORED,NORED,NORED)
        CALL MAX (3,FIN,H,DUM,NORED,NORED,1)
```

```
      DO 1520 I=1,NORED
      FINEW(I,NORE1) = -DUM(I,1) *S
1520  CONTINUE
      C(NORE1,1) =1.0-OB(NORE1)
      CALL MAX(3,FINEW ,C,A,NORE1,NORE1,1)
      NORED=NORE1
      DO 400 I=1,NORED
      DO 400 J=1,NORED
      FIN(I,J) = FINEW(I,J)
400   CONTINUE
C.
C.  FROM THE SET OF COEFFICIENTS A(I) SOLVE THE LIFE TIME
C.
502   SUM0.0
      WRITE(PR,7918)  (A(IT,1) ,IT=1,NORED)
7918  FORMAT('   *****   A **',6(F9.4,1X))
      J=0
      SUM=SUM+A(1 ,1)
      IF(SUM-1.0)  505,510,510
501   SUM=SUM+A(1,1)
      IF( NORED-1) 500,500,513
513   DO 500 I=2,NORED
      RJ=J
      SUM1=RJ**(I-1)
      SUM=SUM+A(I,1)*J**(I-1)
500   CONTINUE
      IF(SUM-1.0) 505,510,510
 505  J=J+1
      IF(J-60) 501,501,510
510   LIFE=J+1
      LIFEN=LIFE-ITSK
      TLIFE=LIFEN/TCNST
      WRITE(PR,7900)LIFEN
7900   FORMAT('  **  **  **  LIFE= ',I5, 'SHOCK')
C.
C.  USING NEWTON'S METHOD TO SOLVE TR
C.  CFW CF/CW
C.
      IF(LIFE -2)   550,560,565
550   TR=0.0
      GO TO 700
560   TR=1.0/(CFW-1.0)
      GO TO 700
565   IF(LIFE-3) 570,570,580
570   TR=(1.0 + SQRT(2.0*CFW-1))/(CFW-1)
      GO TO 700
580   IF(LIFE-60)590,567,567
567   TR=9999.0
      GO TO 700
590   TR=LIFE*2.0
```

```
      ICODP=0
      LIF1=LIFE-1
      LIF2=LIFE-2
      FP=1.0
594   FAT=1.0
      SUM=1.0
      TERM=1.0
      DO 591 I=1,LIF1
      TERM=TERM*TR/I
      SUM=SUM+TERM
591   CONTINUE
      F=CFW*TERM-SUM
      IF(F-0.0)592,700,593
593   TR=TR-1.0
      FP=F
      ICODP=1
      GO TO 594
592   TR=TR-(F/(FP-F))
      LIF1=LIFE-1
      LIF2=LIFE-2
610   FAT=1.0
      SUM=1.0
      TERM=1.0
      DO 600 I=1,LIF2
      TERM=TERM*TR/I
      SUM=SUM+TERM
600   CONTINUE
      DF=CFW*TERM-SUM
      TERM=TERM*TR/LIF1
      SUM=SUM+TERM
      F=CFW*TERM-SUM
      TRN=TR-F/DF
      IF(ABS(TRN-TR)-0.001)   700,700,690
690   TR=TRN
      GO TO 610
700   CONTINUE
      TR=TR*TPD/TCNST
      TR=TR-TPD*ICT
      WRITE(PR,710)  IDE,ITSK,OB(ICT)
710   FORMAT(LH ,/, ' AT TEST POINT',I5, 2X,
     1'THE DEVICE TOTAL RECEIVED',I5,2X,
     2'SHOCKS AND THE MEASUREMENT IS',F15.5)
      WRITE(PR,715)   TLIFE, TR
715   FORMAT(' THE ESTIMATED LIFE TIME IS ',F9.3,'  THE REPLACE TIME IS
     1 ',F7.2)
      IF(TR-TPD) 720,720,730
C.
C.  REPLACE DEVICE
C.
720   WRITE(PR,722)
```

```
722     FORMAT(1H+,T75,'LATER.  WE WILL REPLACE IT NOW')
        IDESN(IDE)=3
        GO TO 3005
C.
C.  CONTINUE USING THE DEVICE
C.
730     WRITE (PR,732)
732     FORMAT(1H+,T75,'LATER.  WE WILL CONTI. USE IT')
        IDESN(IDE)=2
        GO TO 3006
4000    CONTINUE
        CALL EXIT
        END
```